

Fluctuation conductivity in layered d -wave superconductors near critical disorder

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Abstract. We consider the fluctuation conductivity in the critical region of a disorder induced quantum phase transition in layered d -wave superconductors. We specifically address the fluctuation contribution to the system's conductivity in the limit of large (quasi-two-dimensional system) and small (quasi-three-dimensional system) separation between adjacent layers of the system. Both in-plane and c -axis conductivities were discussed near the point of insulator-superconductor phase transition. The value of the dynamical critical exponent, $z = 2$, permits a perturbative treatment of this quantum phase transition under the renormalization group approach. We discuss our results for the system conductivities in the critical region as function of temperature and disorder.

PACS. 74.40.+k Fluctuations (noise, chaos, nonequilibrium superconductivity, localization, etc.) – 73.43.Nq Quantum phase transitions

1 Introduction

A phase transition at $T = 0$ is usually addressed as a quantum phase transition (QPT) [1]. In general, QPT's are driven by quantum fluctuations controlled by a non-thermal parameter, namely by impurities, pressure or magnetic fields [2]. In the case of high temperature superconductors (HTSC) a phase transition can be driven both by disorder or doping [3,4]. As function of disorder the transition is observed for magnetic and nonmagnetic impurities, when the impurity concentration is high enough to destroy the phase coherence in the system. As function of doping the standard phase diagram of HTSC presents two possible QPT's, corresponding to the end points of the superconducting region, one in the underdoped region and the other one in the overdoped region [5,6]. In the underdoped limit the transition is of superconductor-insulator type [3], whereas in the overdoped limit the transition is of superconductor-normal state (metal) type [7].

Modelling high temperature superconductors (HTSC) as two dimensional (2D) systems can be justified by the presence of CuO_2 planes in their structure. However, when transport properties in the critical region around the superconducting phase transition are considered, the theoretical approach has to take into account the quasi-2D nature of the system, as it is known that an important contribution to the conductivity is given by unusual strong fluctuations. Different from metallic superconduc-

tors (MS), HTSC present a strong anisotropy, leading to differences in the temperature dependence of the transverse and in-plane resistivities, a property which is hard to explain based on the conventional theory of Fermi liquids [8]. With this in mind, the simplest way to include the third dimension of the system (c -axis) is to consider a layered structure in which adjacent CuO_2 are coupled through a tunnelling like term. In the presence of disorder the role of fluctuations increases, their effect on the transport properties being very important even for the case of MS [9]. In HTSC, fluctuations are amplified respect to the MS case, their effects being of main importance as transport along c -axis is considered [10], scattering by virtual Cooper pairs increasing the transverse resistivity, in contrast with the usual effect observed in isotropic MS. The analysis of the fluctuations effects in superconducting materials can not distinguish the general symmetry of the order parameter, which is of s -wave type in MS and of d -wave type in HTSC [11,12]. The differences between the two types of symmetry reduce to a constant numerical prefactor, which from the experimental point of view is hard to examine. A more important role is played by the system dimensionality [13]. However, the analysis of the fluctuation conductivity in systems with a p -wave symmetry of the order parameter such as Sr_2RuO_4 [14], reveals the possibility of tracing the pair symmetry in such systems [15].

QPT's are fundamentally different from finite temperature phase transitions as a dynamical critical exponent, z , needs to be considered in order to apply the scaling

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theory to quantum criticality [16]. For the case of HTSC the idea of quantum criticality was largely explored, indications of a 2D superconductor to insulator phase transition with a $z = 1$ critical exponent in the underdoped regime, and a three dimensional (3D) superconductor to normal state phase transition with $z = 2$ in the overdoped regime being identified [5,6]. The disorder induced QPT in d -wave HTSC was analyzed both at $T = 0$ [17] and at finite temperature [18], considering a two dimensional (2D) system and a dynamical critical exponent $z = 2$. For the 2D case, a detailed discussion of the in-plane conductivity was done by Herbut [17] at $T = 0$ and by Dalidovich and Phillips [19,21], with the main conclusion that the dc conductivity will have a *nonuniversal* singular part at the transition point in the quantum critical regime.

In this work we will analyze the disorder induced QPT in 3D d -wave HTSC considering a layered structure of the system. Such an analysis will give us the possibility to study the dimensional crossover of the system as function of the interlayer distance (s). Despite the fact that a layered system is by definition an anisotropic 3D system, at large interlayer distances ($s \rightarrow \infty$) the system approaches a quasi 2D-structure, such that by varying s between 0 and ∞ we basically interpolate between 3D and 2D. In the critical region fluctuation effects dominate the in-plane and c -axis conductivities, which in our calculation are considered of the Aslamazov-Larkin [9] form. On the other hand, for a quasi-3D system, at small separation between adjacent layers, the conductivity conserve the nonuniversal behavior in the quantum critical regime observed in the 2D case [21].

The paper is organized as follows. Section 2 presents our model for the fluctuation propagator in the presence of disorder for the case of a layered d -wave superconductor. Section 3 presents the general equations of the renormalization group approach along with their solutions for two particular regimes, namely, the quantum disorder (QD) and quantum critical (QC) regimes. Section 4 presents analytical solutions for the in-plane and c -axis conductivities for the case of large and small interlayer separation in both QD and QC regimes. Finally, we give our conclusions.

2 Fluctuation propagator in layered systems

We consider that a layered d -wave superconductor in the presence of disorder can be described by a BCS type Hamiltonian with an additional term corresponding to a random interaction potential

$$\begin{aligned} \mathcal{H} = & \sum_{\mathbf{k}, \sigma} \varepsilon(\mathbf{k}) a_{\sigma}^{\dagger}(\mathbf{k}) a_{\sigma}(\mathbf{k}) \\ & - \frac{1}{2} \sum_{\sigma, \sigma'} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} V(\mathbf{k}, \mathbf{k}') a_{\sigma}^{\dagger}(\mathbf{k}_{+}) a_{\sigma'}^{\dagger}(-\mathbf{k}_{-}) a_{\sigma'}(-\mathbf{k}'_{-}) a_{\sigma}(\mathbf{k}'_{+}) \\ & + \sum_i \int d\mathbf{r}_{\parallel} U_i(\mathbf{r}_{\parallel}) a_{\sigma}^{\dagger}(\mathbf{r}_{\parallel}, i) a_{\sigma}(\mathbf{r}_{\parallel}, i), \end{aligned} \quad (1)$$

where $\mathbf{k}_{\pm} = \mathbf{k} \pm \mathbf{q}/2$ and $a_{\sigma}(\mathbf{r}_{\parallel}, i)$ represents the annihilation operator of an electron with spin σ in the i th layer

of the system. We assume that the electronic spectrum in the layered system has the form

$$\varepsilon(\mathbf{k}) = \varepsilon(\mathbf{k}_{\parallel}) + J \cos(k_z s) - E_F, \quad (2)$$

where $\mathbf{k} \equiv (\mathbf{k}_{\parallel}, k_z)$, $\mathbf{k}_{\parallel} \equiv (k_x, k_y)$, and J is the effective hopping energy between two adjacent layers situated at a distance s . The attractive interaction leading to superconductivity, $V(\mathbf{k}, \mathbf{k}')$, is assumed to be separable

$$V(\mathbf{k}, \mathbf{k}') = |g| f(\mathbf{k}) f(\mathbf{k}'), \quad (3)$$

with $f(\mathbf{k}) = [\cos(k_x a) - \cos(k_y a)] h_d(k_z s)$ for the case of d -wave symmetry, $h_d(k_z s)$ being a function reflecting the z axis dispersion of the system. The life time of the quasiparticle, τ , is introduced assuming the Born approximation for the scattering potential and that the random potential obeys the Gaussian ensemble

$$\begin{aligned} \overline{u_i(\mathbf{r}_{\parallel})} &= 0 \\ \overline{u_i(\mathbf{r}_{\parallel}) u_j(\mathbf{r}'_{\parallel})} &= \frac{1}{2\pi N(0)\tau} \delta_{ij} \delta(\mathbf{r}_{\parallel} - \mathbf{r}'_{\parallel}), \end{aligned} \quad (4)$$

where the overline denotes the random average and $N(0)$ the density of states at the Fermi surface. Under such assumptions the quasiparticle Green's function is given by

$$G(\mathbf{k}, i\omega_n) = \frac{1}{i\omega_n \left[1 + (2\tau|\omega_n|)^{-1} \right] - \varepsilon(\mathbf{k})}, \quad (5)$$

where $\omega_n = (2n+1)\pi T$ is the standard fermionic Matsubara frequency.

Following the original procedure introduced by Aslamazov and Larkin [9] the fluctuation propagator in the presence of disorder can be calculated as

$$K^{-1}(\mathbf{q}, i\omega_n) = \frac{1}{gN(0)} - \Pi(\mathbf{q}, i\omega_n), \quad (6)$$

where in this definition $\omega_n = 2n\pi T$ denotes a bosonic Matsubara frequency and

$$\Pi(\mathbf{q}, i\omega_n) = T \sum_{\omega_m} \sum_{\mathbf{k}} |f(\mathbf{k})|^2 G(\mathbf{k}, i\omega_m) G(\mathbf{q} - \mathbf{k}, i\omega_n - i\omega_m). \quad (7)$$

Note that the summation over momenta \mathbf{k} in equation (7) has to be done assuming a cylindrical symmetry of the Fermi surface to ensure that the correct symmetry imposed by the layered structure of the system is well considered. The calculation of the fluctuation propagator is straightforward

$$K(\mathbf{q}, i\omega_n) = \frac{1}{N(0) \left[\mu_0(T, D) + \gamma|\omega_n| + \xi_0^2 q_{\parallel}^2 + 4 \left(\frac{\xi_{c0}}{s} \right)^2 \sin^2 \left(\frac{q_z s}{2} \right) \right]}, \quad (8)$$

where $\mu_0(T, D) = 2\pi^2(T\tau)/3 + (D - D_c)/D_c$ gives the distance to the phase transition point ($D = 1/\tau$ represents

the disorder variable and $D_c = 1.76 T_{c0}$, T_{c0} being the superconducting critical temperature in a clean system), $\gamma \simeq \tau$, $\xi_0^2 = l^2/2$ ($l = v_F \tau$), $\xi_{c0}^2 = (J\tau s)^2/2$. According to the Thouless criterion the phase transition occurs at $K^{-1}(0, 0) = 0$, meaning that in our case it can be driven both by disorder or temperature. At the quantum critical point ($T = 0$) the phase transition is induced by disorder, $\mu_0(T = 0, D = D_c) = 0$. On the other hand, in the weak disorder limit, $D < D_c$, the phase transition occurs at a finite temperature such that $\mu_0(T = T_c, D) = 0$. The layered structure of the considered system is well reflected in the form of the fluctuation propagator $K(\mathbf{q}, i\omega_n)$ as it is easy to see from equation (8). At large interlayer separation, $s \rightarrow \infty$, the last term in the denominator of the right hand side of the equation becomes small and the fluctuation propagator corresponding to a quasi-2D system is recovered. When the interlayer separation is small, $s \rightarrow 0$, it is easy to see that the fluctuation propagator will have the form corresponding to the 3D case. The crossover between 2D and 3D is then possible as a function of the interlayer separation, s .

The general form of the action describing the phase fluctuations in the critical region of the phase transition can be obtained following the standard procedure [22] to decouple the standard BCS action as

$$S_{\text{eff}} = \sum_q \phi^\dagger(q) K^{-1}(q) \phi(q) + \frac{u}{4} \sum_{q_1 \dots q_4} \phi(q_1) \dots \phi(q_4) \delta(q_1 + q_2 + q_3 + q_4), \quad (9)$$

where $\phi(q)$ are the fluctuation field operators, and u measure the interaction between fluctuations. In the above equation, $q \equiv (\mathbf{q}, \omega_n)$ and

$$\sum_q \dots \rightarrow k_B T \sum_n \int \frac{d^d q}{(2\pi)^d},$$

d being the system dimensionality.

3 Renormalization-group analysis of the transport properties

For a detailed analysis of the transport properties in the critical region of a QPT we will use the general formalism of the renormalization group approach introduced by Hertz [16] and developed lately by Millis [23]. The main idea in the transport properties evaluation is that according to the renormalization group approach the conductivity obeys the scaling relation [20, 21]

$$\sigma_{\alpha\beta}(\mu_0, T, \omega, u) = e^{(d-2)l^*} \sigma_{\alpha\beta}^* [T(l^*), \omega(l^*), u(l^*)], \quad (10)$$

where the scaling l^* is defined such as the renormalization procedure stop at l^* given by $\mu(l^*) = 1$. The asterisk denotes that the conductivity is considered at the fixed point, i.e., for $l = l^*$. For the most general case the

renormalization group equations corresponding to the action given by equation (9) can be obtained performing the standard scaling $k = k'/b$ and $\omega_n = \omega'_n/b^z$ ($b = \ln(l)$) as

$$\frac{d\Gamma(l)}{dl} = (z - 2)\Gamma(l), \quad (11a)$$

$$\frac{dT(l)}{dl} = z T(l), \quad (11b)$$

$$\frac{d\mu(l)}{dl} = 2\mu(l) + \frac{K_d \Gamma(l) u(l)}{\exp\left[\frac{\Gamma(l)}{T(l)} (\Lambda^2 + \mu(l))\right] - 1}, \quad (11c)$$

$$\frac{du(l)}{dl} = [4 - (d + z)] u(l) - \frac{2K_d \Lambda^d T^2(l)}{4T(l) \sinh^2\left[\frac{\Gamma(l)}{T(l)} (\Lambda^2 + \mu(l))\right]} u^2(l), \quad (11d)$$

where z denotes the dynamical critical exponent, $\Gamma = 1/\gamma$, K_d is a dimension dependent constant, and $\Lambda(l)$ is a momenta cutoff. The set of equations (11) admit an unstable Gaussian fixed point at $\Gamma = T = \mu = u = 0$. In the following we will consider separately the quantum disorder and quantum critical regimes, as the renormalization group equations lead to different solutions near the Gaussian fixed point.

3.1 Quantum disorder regime

In the quantum disorder regime ($\mu_0 \gg T$) for the case $d = 3$ and $z = 2$ the renormalization group equations can be solved relatively easy. The first two equations, (11a) and (11b), lead to simple solutions, namely $\Gamma(l) = \text{const.}$ and $T(l) = T e^{2l}$, respectively. Because $d + z > 4$, u is irrelevant and as a consequence the second term in equation (11c) can be neglected leading to $\mu(l) = \mu_0 e^{2l}$. Accordingly, the renormalization procedure will be stopped at

$$l^* = \frac{1}{2} \ln \frac{1}{\mu_0}, \quad (12)$$

μ_0 being the initial value of the distance to the phase transition point. The system temperature at the fixed point becomes

$$T(l^*) = \frac{T}{\mu_0}. \quad (13)$$

Note that a solution for the interaction term can be obtained as $u(l^*) \sim \sqrt{\mu_0}$, a result which is in agreement with the initial discussion of the interaction irrelevance in the $d = 3$, $z = 2$ case.

3.2 Quantum critical regime

The quantum critical regime is defined by $\mu_0 \ll T$. In this case the integration of the renormalization group equations is more complicated as the critical region

around the phase transition consists of two different domains, associated to quantum and classical effects. The crossover between the two domains is characterized by $\tilde{l} = [\ln(1/T)]/2$, such that $T(\tilde{l}) = 1$. The integration over the scaling has to be split in two domains, corresponding to quantum ($l < \tilde{l}$) and classical ($\tilde{l} < l < l^*$) behavior. Consider now equation (11c), which will lead to the general equation for the scaling l^* at which the renormalization procedure is stopped ($\mu(l^*) = 1$). An approximate solution of this equation can be obtained in two steps. First we introduce a new scaling variable, $l' = l - \tilde{l}$, and secondly we neglect $\mu(l)$ in the exponential term occurring in the right hand side of the equation. The equation can be rewritten as

$$\frac{d\mu(l')}{dl'} = 2\mu(l') + \frac{K_3 u}{\exp[e^{-2l'}] - 1}, \quad (14)$$

and admits the following solution

$$\mu(l') = \frac{K_3 u}{2} \ln \left(\frac{e - 1}{\exp[e^{-2l'}] - 1} \right) - \frac{K_3 u}{2} (1 - e^{-2l'}). \quad (15)$$

Without loss of generality we assumed in both equations (14) and (15) that $A^2 = 1$. The renormalization procedure will be stopped at l^* satisfying the following condition

$$\frac{2}{K_3 u} = 1 - e^{2(l^* - \tilde{l})} + 2(l^* - \tilde{l}) e^{2(l^* - \tilde{l})}. \quad (16)$$

Finding an exact analytical solution for the above equation is not possible, so we chose to solve the equation iteratively, the solution within double logarithmic accuracy being of the form

$$l^* = \frac{1}{2} \ln \left[\frac{2}{T K_3 u \ln[2/(K_3 u)]} \right]. \quad (17)$$

The corresponding renormalized temperature is

$$T(l^*) = \frac{2}{K_3 u \ln[2/(K_3 u)]}. \quad (18)$$

In the following we will turn our attention to the system's conductivity. We will evaluate the main contribution to the conductivity in the framework of the renormalization group approach.

4 Fluctuation conductivity in layered systems

The main contribution to the system conductivity will be calculated following Aslamazov and Larkin [9] based on the Kubo formula. The layered structure of the system is associated to the conductivity anisotropy in the system and accordingly we will have to estimate different contributions for the in-plane and c -axis conductivities. Following Varlamov *et al.* [10] the main contribution to the conductivity tensor can be calculated as

$$\sigma_{\alpha\beta}^* = - \lim_{\omega \rightarrow 0} \frac{1}{i\omega} [Q_{\alpha\beta}]^R(\omega), \quad (19)$$

where $Q_{\alpha\beta}(\omega)$ represents the electromagnetic response operator which contribute to the fluctuation conductivity of the layered system. In equation (19) the subscripts (α, β) represents the polarization directions and R denotes the retarded part of the operator. A diagrammatic evaluation of the electromagnetic response operator (see Ref. [10]) leads to the following general form of the conductivity

$$\sigma_{\alpha\beta}^* = \frac{2\pi\xi_0^4 m^2}{R_Q T(l^*)} \int \frac{d\omega}{\sinh^2\left(\frac{\omega}{2T(l^*)}\right)} \times \int \frac{d^2 q_{\parallel}}{(2\pi)^2} \int_{-\pi/s}^{\pi/s} \frac{dq_z}{2\pi} v_{\alpha} v_{\beta} [Im K^R(\mathbf{q}, \omega)]^2, \quad (20)$$

where $R_Q = \pi\hbar/(2e^2)$, and $v_{\alpha} = [\partial\varepsilon(p)/\partial p_{\alpha}]$. In equation (20) the initial integration over momenta was considered based on the cylindrical symmetry of the system. Accordingly, the in-plane and respectively the c -axis conductivities become

$$\sigma_{\parallel}^* = \frac{8\pi\gamma\xi_0^4 T(l^*)}{R_Q \mu_0^3} \int d\omega f\left(\frac{\mu_0\omega}{2\gamma T(l^*)}\right) \int \frac{d^2 q_{\parallel}}{(2\pi)^2} \times \int_{-\pi/s}^{\pi/s} \frac{dq_z}{2\pi} \frac{q_{\parallel}^2}{\left\{ \left[1 + \frac{\xi_0^2}{\mu_0} q_{\parallel}^2 + \frac{4}{\mu_0} \left(\frac{\xi_{c0}}{s}\right)^2 \sin^2\left(\frac{q_z s}{2}\right) \right]^2 + \omega^2 \right\}^2} \quad (21)$$

and

$$\sigma_c^* = \frac{8\pi\gamma\xi_0^4 T(l^*)}{R_Q \mu_0^3} \int d\omega f\left(\frac{\mu_0\omega}{2\gamma T(l^*)}\right) \int \frac{d^2 q_{\parallel}}{(2\pi)^2} \times \int_{-\pi/s}^{\pi/s} \frac{dq_z}{2\pi} \frac{m^2 J^2 s^2 \sin^2(q_z s)}{\left\{ \left[1 + \frac{\xi_0^2}{\mu_0} q_{\parallel}^2 + \frac{4}{\mu_0} \left(\frac{\xi_{c0}}{s}\right)^2 \sin^2\left(\frac{q_z s}{2}\right) \right]^2 + \omega^2 \right\}^2}, \quad (22)$$

where $f(x) = x^2/[\sinh^2(x)]$. A similar way to investigate the system conductivity was done in reference [15], with the specification that the final result for the in-plane conductivity is obtained replacing the original layered symmetry of the system with an isotropic 3D one. However, our approach is different, as the original cylindrical symmetry of the system is conserved in the conductivity calculation. The analytical structure of the integrand in both equations (21) and (22) allows us to distinguish between the QD and QC regimes.

4.1 Quantum disorder regime

In the quantum disorder regime, $\mu_0 \gg T$, the main contribution in the integration over the frequency variable in both parallel and c -axis conductivities is associated to the $\omega = 0$ point. Based on this approximation the two

conductivities can be calculated as

$$\sigma_{\parallel}^* = \frac{2\pi^2\gamma^2 T^2(l^*)}{9R_Q s \mu_0^2} \frac{1 + \frac{2}{\mu_0} \left(\frac{\xi_{c0}}{s}\right)^2}{\left\{ \left[1 + \frac{2}{\mu_0} \left(\frac{\xi_{c0}}{s}\right)^2\right]^4 - \frac{4}{\mu_0^2} \left(\frac{\xi_{c0}}{s}\right)^4 \right\}^{3/2}}, \quad (23)$$

and

$$\sigma_c^* = \frac{2\pi^2\gamma^2 T^2(l^*)}{9R_Q s \mu_0^3} \frac{s^2 \xi_0^2 m^2 J^2}{\left\{ \left[1 + \frac{2}{\mu_0} \left(\frac{\xi_{c0}}{s}\right)^2\right]^4 - \frac{4}{\mu_0^2} \left(\frac{\xi_{c0}}{s}\right)^4 \right\}^{3/2}}. \quad (24)$$

A better understanding of the in-plane and c -axis conductivities is achieved in the limit of large and small separation between adjacent layers of the system.

4.1.1 Large interlayer separation, $s \rightarrow \infty$

For the large interlayer separation case, $s \rightarrow \infty$, which from the dimensional point of view approaches a quasi 2D system, the system's conductivities evaluated at the fixed point are

$$\sigma_{\parallel} = \frac{2\pi^2\gamma_1^2 T^2}{9R_Q s \mu_0^6} \quad (25)$$

and

$$\sigma_c = \frac{2\pi^2\gamma_1^2 \xi_0^2 m^2 J^2 s T^2}{9R_Q \mu_0^7}. \quad (26)$$

Note that equations (26) and (25) were obtained using the renormalized values for the system's temperature in the QD regime. The in-plane and c -axis conductivities have both the same temperature dependence but different disorder dependence close to the QPT point.

4.1.2 Small interlayer separation, $s \rightarrow 0$

In the case of small interlayer separation, $s \rightarrow 0$, the system approaches a quasi 3D system. A simple calculation of the system's conductivities at the fixed point leads to

$$\sigma_{\parallel} = \frac{\pi^2\gamma_1^2}{596R_Q \xi_{c0}} \frac{T^2}{\mu_0^{11/2}} \quad (27)$$

and

$$\sigma_c = \frac{\pi^2\gamma_1^2 \xi_0^2 m^2 J^2 s^4}{36R_Q \xi_{c0}^3} \frac{T^2}{\mu_0^{11/2}}. \quad (28)$$

Different from the large interlayer separation case, in the quasi-3D case the temperature and disorder dependence of the two conductivities is the same.

4.2 Quantum critical regime

In the QC regime, $\mu_0 \ll T$, we approximate $f(x) \rightarrow 1$, as in this situation $x \ll 1$. The analytical forms of the in-plane and c -axis conductivities are obtained as

$$\sigma_{\parallel} = \frac{\pi\gamma T}{2R_Q s \mu_0} \frac{1}{\left\{ \left[1 + \frac{2}{\mu_0} \left(\frac{\xi_{c0}}{s}\right)^2\right]^4 - \frac{4}{\mu_0^2} \left(\frac{\xi_{c0}}{s}\right)^4 \right\}^{1/2}} \quad (29)$$

and

$$\sigma_c = \frac{\pi\gamma T \xi_0^2 m^2 J^2 s}{2R_Q \mu_0^2} \frac{1 + \frac{2}{\mu_0} \left(\frac{\xi_{c0}}{s}\right)^2 - \left\{ \left[1 + \frac{2}{\mu_0} \left(\frac{\xi_{c0}}{s}\right)^2\right]^4 - \frac{4}{\mu_0^2} \left(\frac{\xi_{c0}}{s}\right)^4 \right\}^{1/2}}{\frac{4}{\mu_0^2} \left(\frac{\xi_{c0}}{s}\right)^4 \left\{ \left[1 + \frac{2}{\mu_0} \left(\frac{\xi_{c0}}{s}\right)^2\right]^4 - \frac{4}{\mu_0^2} \left(\frac{\xi_{c0}}{s}\right)^4 \right\}^{1/2}}. \quad (30)$$

In the following we will consider a simplified form of the two obtained equations in the limit of large and small interlayer separation in order to discuss the temperature and disorder dependence of the conductivity.

4.2.1 Large interlayer separation, $s \rightarrow \infty$

For the large interlayer separation between two adjacent planes of the layered structure the in-plane and c -axis conductivities can be approximated as

$$\sigma_{\parallel} = \frac{4\pi\gamma_1}{R_Q s \left(K_3 u \ln \frac{2}{K_3 u}\right)^3} \frac{1}{\mu_0 T^2} \quad (31)$$

and

$$\sigma_c = \frac{2\pi\gamma_1 \xi_0^2 m^2 J^2 s}{R_Q \left(K_3 u \ln \frac{2}{K_3 u}\right)^3} \frac{1}{\mu_0^2 T^2}. \quad (32)$$

As in the QD regime, the temperature dependence of the two conductivities at large separation between adjacent layers of the system is the same, however, the disorder dependence of the two conductivities differs.

4.2.2 Small interlayer separation, $s \rightarrow 0$

Let us consider now the small interlayer separation case, resembling the quasi 3D-system. The in-plane and c -axis conductivities are obtained as

$$\sigma_{\parallel} = \frac{2\pi\gamma_1}{R_Q \xi_{c0} \left(K_3 u \ln \frac{2}{K_3 u}\right)^3} \frac{1}{\mu_0^{1/2} T^2} \quad (33)$$

and

$$\sigma_c = \frac{\pi\gamma_1 \xi_0^2 m^2 J^2 s^4}{R_Q \xi_{c0}^3 \left(K_3 u \ln \frac{2}{K_3 u}\right)^3} \frac{1}{\mu_0^{1/2} T^2}. \quad (34)$$

Once again for the quasi-3D case the temperature and disorder dependence of the two conductivities is similar.

5 Conclusions

In conclusion we presented a detailed analysis of the in-plane and c -axis conductivities close to the QPT points identified in d -wave superconductors. We considered that the system is well described by a layered structure of coupled planes, a model which allowed us to analytically obtain the two components of the system's conductivity. For our model, the system dimensionality is fixed at $d = 3$ despite the fact that large interlayer separation resemble a quasi-2D dimensional system; at small interlayer separation the anisotropic 3D system is recovered. The system's dimensionality, $d = 3$, together with the dynamical critical exponent, $z = 2$, makes the results of our calculation suitable to describe the QPT in the overdoped region of HTSC phase diagram. However, the possibility to interpolate between quasi-2D and 3D systems due to the layered structure of the system can provide a better understanding of the disorder induced phase transition in HTSC in both underdoped and overdoped regions of the phase diagram.

Our estimation of the system's conductivity was based on the renormalization group approach. The analysis of the possible QPT's in d -wave superconductors was done by Schneider [5,6] based on a detailed revision of experimental data for correlation length, magnetic penetration depth, specific heat and resistivity. His conclusions are different for the two QPT points observed in the underdoped and overdoped region of the phase diagram. In the underdoped region a quantum superconductor-insulator transition was identified, whose characteristics are $d = 2$ and $z = 1$. On the other hand, in the overdoped region a superconductor-normal state transition was identified with $d = 3$ and $z = 2$. The choice of our system symmetry and implicitly the form of our fluctuation propagator forced us to consider a fix dimensionality of the system, namely $d = 3$, meaning that the point $d = 2$ is not accessible to our approximation. We also considered $z = 2$ according to the experimental data. However, as we can vary the interlayer separation distance we were able to approach a quasi-2D system and show that a possible QPT is present even in this situation, both as function of disorder or temperature. In the $d = 3$, $z = 2$ limit the analysis of the renormalization group equations is straightforward, leading to a Gaussian fixed point. In this situation the quadratic term in the system action, describing direct interaction between fluctuations, is irrelevant, meaning that for a certain range of temperature and disorder around the fixed point we can calculate the system's conductivity in the perturbative renormalization group.

We considered two different regimes in our approximations, namely the quantum disorder regime, $T \ll \mu_0$, and the quantum critical regime, $T \gg \mu_0$. Analytical results were presented for both type of conductivities in two different limits, for large and small separation between the

component layers of the system. As a general rule the temperature dependence of the these conductivities is similar for all situations in our calculation, as long as we consider the same regime. On the other hand, for both large and small interlayer separation limits, when we lower the temperature the system resistivity should be nonmonotonic, a dip at a temperature of the order $T \sim \mu_0$ being present. A similar result was reported in reference [21] for the case $d = 2$ and $z = 2$, which actually can be seen as a extreme limit of our calculation. The situation changes as we investigate the disorder dependence of the conductivities. At small separation between adjacent layers, when we approach a quasi-3D system both in-plane and c -axis conductivities diverge at the critical disorder point following the same power law, whatever we are in the quantum disorder or quantum critical regime. At large separation, when the system resemble a quasi-2D one, the behavior of the in-plane and c -axis conductivities at the QCP are different as function of disorder. Despite the fact that both conductivities diverge at the critical disorder point following power laws, the divergence observed for the c -axis conductivity is stronger.

The case of a pure 2D system was previously considered by Dalidovich and Phillips [21] at finite temperature, and by Herbut at $T = 0$ [17]. Our result for the in-plane conductivity, at large separation between adjacent layers of the system, is similar to the one obtained in reference [21]. Some differences occurs due to the different system dimensionality and a difference between the form of the quasiparticle propagator consider here and in reference [21]. However, the main result for the two dimensional case, which states that the in-plane conductivity in the quantum critical regime is non universal and increase as function of temperature when the QCP ($T \rightarrow 0$) is approached was also proved in our calculation. Moreover, our investigation presents qualitative results also for the quasi-3D case, which is relevant for the case of d -wave superconductors, where, despite the fact that conduction in the superconducting phase is attributed to the CuO planes, the system is a 3D one.

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